Title of your IAU Symposium
Proceedings IAU Symposium No. xxx, 2010
A.C. Editor, B.D. Editor & C.E. Editor, eds.

DOI: 00.0000/X000000000000000X

Flux-transport and mean-field dynamo theories of solar cycles

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Abstract. We point out the difficulties in carrying out direct numerical simulation of the solar dynamo problem and argue that kinematic mean-field models are our best theoretical tools at present for explaining various aspects of the solar cycle in detail. The most promising kinematic mean-field model is the flux transport dynamo model, in which the toroidal field is produced by differential rotation in the tachocline, the poloidal field is produced by the Babcock–Leighton mechanism at the solar surface and the meridional circulation plays a crucial role. Depending on whether the diffusivity is high or low, either the diffusivity or the meridional circulations provides the main transport mechanism for the poloidal field to reach the bottom of the convection zone from the top. We point out that the high-diffusivity flux transport dynamo model is consistent with various aspects of observational data. The irregularities of the solar cycle are primarily produced by fluctuations in the Babcock–Leighton mechanism and in the meridional circulation. We summarize recent work on the fluctuations of meridional circulation in the flux transport dynamo, leading to explanations of such things as the Waldmeier effect.

Keywords. Sun: activity, Sun: dynamo, sunspots

1. In defence of kinematic mean-field models

We believe that the solar magnetic fields are generated by the interactions between the velocity field $\mathbf{v}$ and the magnetic field $\mathbf{B}$ within the Sun's convections zone. These interactions are described by the following MHD equations:

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}.\nabla)\mathbf{v} = -\frac{1}{\rho}\nabla\left(p + \frac{B^2}{8\pi}\right) + \frac{(\mathbf{B}.\nabla)\mathbf{B}}{4\pi\rho} + \mathbf{g} + \nu\,\nabla^2\mathbf{v}, \quad (1)$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \lambda\,\nabla^2\mathbf{B}. \quad (2)$$

Here ρ is density, p is pressure, $\mathbf{g}$ is gravitational field, ν is kinematic viscosity and λ is magnetic diffusivity.

There are two possible approaches to the solar dynamo problem for explaining the generation of solar magnetic fields.

- Direct numerical simulation or DNS, in which one puts all the MHD equations in the computer and solves them.
- The kinematic approach, in which the velocity field is given and one solves only the equation (2) for the magnetic field.

Historically, solar dynamo theory developed by following the kinematic approach. Since the turbulence in the solar convection zone is crucial for the dynamo action, the kinematic theory has to be of the nature of a mean field theory in which we average over fluctuations around the mean (Parker 1955; Steenbeck, Krause & Rädler 1966). We can split both the velocity field and the magnetic field into mean and fluctuating parts, i.e.

$$\mathbf{v} = \overline{\mathbf{v}} + \mathbf{v}', \quad \mathbf{B} = \overline{\mathbf{B}} + \mathbf{B}'. \quad (3)$$

Here the overline indicates the mean and the prime indicates the departure from the mean. On substituting (3) in (2), one finds that $\overline{\mathbf{B}}$ obeys an equation more complicated than (2) obeyed by $\mathbf{B}$ — the celebrated dynamo equation (see, for example, Choudhuri 1998; § 16.5). Kinematic mean-field models are based on this dynamo equation.

Those of us who work on kinematic mean-field models of the solar dynamo nowadays often face a question: "Maybe the kinematic mean-field approach was the only possible approach when Parker was developing the subject. But is it not old-fashioned and out-of-date to still work on kinematic mean-field models? Why don't you do DNS?" The first attempt of doing a DNS of the solar dynamo was made quite early — by Gilman (1983). But Gilman's simulations failed to reproduce even the most basic features of the solar cycle, and the reasons for this failure are still not fully understood. Perhaps this early failure discouraged further work on DNS of the solar dynamo for a long time.

Impressive simulations of the geodynamo are now available — beginning with the work of Glatzmaier & Roberts (1995). Even the random reversals of the geomagnetic field had been successfully modelled. Doing a DNS of the solar dynamo is much more challenging for the following reasons.

- The solar convection zone is highly stratified within which quantities like density and pressure vary by several orders of magnitude from the bottom to the top.
- The largest relevant scales (10^6 km) differ from the smallest scales of fibril flux tubes (10^2 km) by 4 orders.

Additionally, a kinematic model has one tremendously important advantage. Within the last few years, helioseismology has provided considerable information about the velocity fields in the Sun's interior and it has now been possible to put these velocity fields measured by helioseismology directly into the kinematic dynamo models, thereby revolutionizing the field. In a DNS, on the other hand, the velocity fields have to be calculated from the basic equations of fluid mechanics and, until one gets the velocity fields correctly, there is no hope of getting the magnetic fields correctly. Of late, several research groups are again working on DNS of the solar dynamo problem (Ghizaru, Charbonneau & Smolarkiewicz 2010; Brown et al. 2010). It is possible that a major breakthrough is around the corner. However, until such a breakthrough takes places, the DNS models are still of rather exploratory nature. If one seeks theoretical explanations of various detailed aspects of the solar cycle, so far the kinematic mean-field model is the only possible approach. Also, we hope that results of kinematic mean-field models will provide useful guidance in the development of DNS models.

2. Flux transport dynamo model

The central idea of solar dynamo theory is that the toroidal and the poloidal components of the magnetic field sustain each other through a feedback loop. It is easy to see how the poloidal field may give rise to the toroidal field. The differential rotation of the Sun is expected to stretch the poloidal field lines in the toroidal direction. Since the differential rotation is strongest in the tachocline at the base of the convection zone, we expect the toroidal field to be primarily produced there. This toroidal field then has to rise through the convection zone due to magnetic buoyancy to produce sunspots. Numerical simulations of this buoyant rise suggested that the magnetic field at the bottom of the convection zone has to be as strong as 10^5 G (Choudhuri & Gilman 1987; Choudhuri 1989; D'Silva & Choudhuri 1993; Fan, Fisher & DeLuca 1993).

The generation of the poloidal field from the toroidal field is a more complicated problem. Historically there have been two influential schools of thought. The original idea of Parker (1955) and Steenbeck, Krause & Rädler (1966) is that turbulence in the

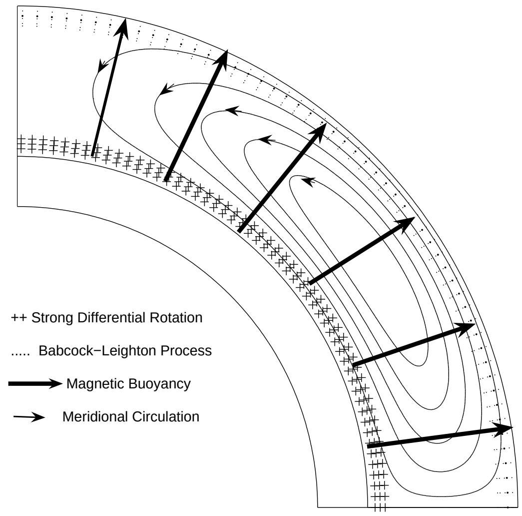

Figure 1. A cartoon explaining how the flux transport dynamo works.

presence of rotation has a preferred helicity and this helical turbulence can twist the toroidal field to produce the poloidal field. This mechanism is often called the α-effect. The alternative idea due to Babcock (1961) and Leighton (1964) is based on the fact the bipolar sunspot pairs on the solar surface appear with a tilt, produced by the action of the Coriolis force on the rising flux tubes (D'Silva & Choudhuri 1993). After the decay of tilted bipolar sunspots, their magnetic fields diffuse around to give rise to a poloidal field. A tilted bipolar sunspot pair can thus be viewed as a conduit for converting the toroidal field to the poloidal field. It forms from the toroidal field and we get the poloidal field after its decay.

Many of the early solar dynamo calculations were based on the α-effect. However, the α-effect can twist the toroidal field only if it has a value not much stronger than the value that would produce equipartition between the energies of turbulence and magnetic field. The equipartition value of the magnetic field at the bottom of the convection zone is expected to be not larger than 10^4 G. When flux rise simulations indicated that the toroidal field may be as strong as 10^5 G, it appeared that the α-effect will not be able to twist such a strong field. Hence the Babcock–Leighton mechanism is invoked in many recent models. Since we see this mechanism operating on the solar surface, we believe this to be the dominant mechanism under normal circumstances. But, during the grand minima when sunspots are absent, the Babcock–Leighton mechanism may not take place. We probably need something like an α-effect to pull the Sun out of a grand minimum. How the Sun comes out of a grand minimum is very poorly understood at the present time. The discussion in this review will be restricted to normal situations when the Babcock–Leighton mechanism is presumably the primary mechanism for generating the poloidal field.

We can consider a dynamo in which the toroidal field is produced by differential rotation in the tachocline and the poloidal field is produced near the surface by the Babcock–Leighton mechanism. Choudhuri, Schüssler & Dikpati (1995), who studied this kind of dynamo, pointed out one possible difficulty. According to the Parker-Yoshimura sign rule (Parker 1955; Yoshimura 1975; Choudhuri 1998, §16.6), a dynamo based solely on these effects will have a poleward propagation, contrary to the observations. However,

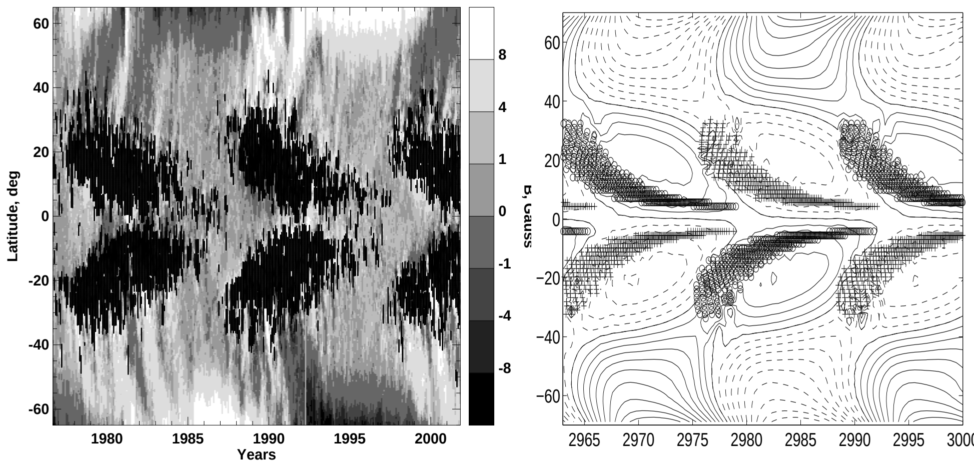

Figure 2. Butterfly diagram of sunspots superposed on the time-latitude plot of B_r. The observational plot is shown on the left. The comparable theoretical plot obtained by the dynamo model of Chatterjee, Nandy & Choudhuri (2004) is on the right.

Choudhuri, Schüssler & Dikpati (1995) showed that a suitable meridional circulation can reverse the propagation direction and can give rise to equator-propagating butterfly diagrams. Dynamo models in which the meridional circulation plays an important role are called flux transport dynamo. Such a dynamo was considered in an early paper by Wang, Sheeley & Nash (1991). First two-dimensional models of the flux transport dynamo were constructed by Choudhuri, Schüssler & Dikpati (1995) and Durney (1995).

Fig. 1 shows a schematic cartoon of the flux transport dynamo. The meridional circulation which is equatorward at the bottom of the convection zone advects the toroidal field produced there. On the other hand, the poleward meridional circulation near the surface advects the poloidal field poleward, which is seen observationally. Fig. 2 shows the butterfly diagram of sunspots in the same plot along with the time-latitude plot of the radial field at the surface. The theoretical plot obtained by Chatterjee, Nandy & Choudhuri (2004) shown on the right has to be compared with the observational plot shown on the left. Most of the calculations of the flux transport dynamo model have been done with single-celled meridional circulation which reach out to the bottom of the convection zone, as shown in Fig. 1. In fact, it is found that certain aspects of observational data can be explained best if the meridional circulation is assumed to penetrate slightly below the bottom of the convection zone (Nandy & Choudhuri 2002; Chakraborty, Choudhuri & Chatterjee 2009). There are some recent indications that the meridional circulation may be more complicated than this and the return flow may be at a shallower depth (Hathaway 2012; Junwei Zhao, private communication). It remains to be seen whether these results get confirmed by other independent studies. Flux transport dynamo models with more complicated meridional circulation are yet to be studied properly.

While the flux transport dynamo model, as sketched in Fig. 1, may not yet be a universally accepted model of the solar cycle, more and more scientists are finding it an attractive model and considerable work has been done on this model in the last few years by different groups around the world. Given the limited scope of this review, it is not possible to cover all the aspects of the flux transport dynamo model currently under investigation. The next two sections provide a rather personal and subjective sampling of topics which are of special interest to this author and on which our group had been

working. Notwithstanding the disclaimer that this is not a comprehensive review of the flux transport dynamo, hopefully readers will get an idea of some of the crucial issues. Some issues connected with the flux transport dynamo not discussed here are discussed by Jiang (2013).

3. Two classes of models and the problem of modelling irregularities of the solar cycle

In the flux transport dynamo model, the toroidal field is produced at the bottom of the convection zone, whereas the poloidal field is produced at the top. We need some transport mechanisms between these two source regions in order for the dynamo to work. As shown in Fig. 1, the toroidal field is transported by magnetic buoyancy from the bottom to the top where the Babcock–Leighton mechanism acts on it to produce the poloidal field. We also need a transport mechanism for bringing the poloidal field from the top to the bottom where differential rotation can act on it to produce the toroidal field. Two classes of models have been worked out in the last few years based on two different dominant transport mechanisms for this. If the turbulent diffusivity of the convection zone is assumed to be sufficiently high to make the diffusion time across the convection zone of order 5 years, then the poloidal field reaches the bottom from the top by diffusion. Such high-diffusivity models have been constructed in Bangalore by Choudhuri and his successive students (Nandy, Chatterjee, Jiang, Karak). On the other hand, Dikpati and her co-workers in Boulder (Charbonneau, Gilman, de Toma) have developed a low-diffusivity model in which the diffusion time scale across the convection zone is of order hundreds of years and the poloidal field is advected from the top to the bottom by the meridional circulation shown in Fig. 1. The differences between these models have been systematically studied by Jiang, Chatterjee & Choudhuri (2007) and Yeates, Nandy & Mckay (2008). Both these models are capable of giving rise to oscillatory solutions resembling solar cycles. However, when we try to study the irregularities of the cycles, the two models give completely different results. We need to introduce fluctuations to cause irregularities in the cycles. In the high-diffusivity model, the fluctuations spread all over the convection zone in about 5 years. On the other hand, in the low-diffusivity model, the fluctuations essentially remain frozen during the cycle period. Thus the behaviours of the two models are totally different on introducing fluctuations.

It may be mentioned that both these models were used a few years ago to predict the strength of the cycle 24 before it had begun. Dikpati & Gilman (2006) used their low-diffusivity model to predict that the cycle 24 will have the peak sunspot number in the range 157–181, making it one of the strongest recorded cycles. On the other hand, Choudhuri, Chatterjee & Jiang (2007) used their high-diffusivity model to predict a peak sunspot number of around 70–80, implying that it will be a weak cycle. While the jury is not completely out yet, the early indications suggest that the prediction from the high-diffusivity model will be much closer to the truth. This lends more credibility to the high-diffusivity model. We list below several other arguments in favour of the high-diffusivity model.

- The diffusivity assumed in the high-diffusivity model is consistent with the simple mixing length estimate $\frac{1}{3}vl$ (Parker 1979, p. 629; Miesch et al. 2012).
- The high diffusivity helps in establishing dipolar parity which the solar magnetic fields seem to have (Chatterjee, Nandy & Choudhuri 2004; Hotta & Yokoyama 2010).
- The high diffusivity keeps the hemispheric asymmetry in solar activity small as observed (Chatterjee & Choudhuri 2006; Goel & Choudhuri 2009).

• The surface flux transport models also require a similar diffusivity near the surface (Wang, Nash & Sheeley 1989).

• The high-diffusivity model reproduces the observed correlation between the polar field at the minimum and the strength of the next cycle (Jiang, Chatterjee & Choudhuri 2007).

The last point is crucial in the prediction of forthcoming cycles. In the work on predicting cycle 24, Choudhuri, Chatterjee & Jiang (2007) had fed the information in the dynamo model that the polar field during the preceding minimum was weak and it is an inevitable consequence of the high-diffusivity model that the next cycle has to be weak as a result of the last point (Jiang, Chatterjee & Choudhuri 2007). Apart from the arguments listed above, we shall see in the next section that the high-diffusivity model is able to explain certain other aspects of observational data (such as the Waldmeier effect) on introducing fluctuations in meridional circulation.

Let us mention here that, apart from meridional circulation and diffusion, there is another possible mechanism for transporting the poloidal field from the top of the convection zone to the bottom: turbulent pumping. Only recently effects of this on the flux transport dynamo have started being studied (Karak & Nandy 2012; Jiang et al. 2012). Since the downward transport time scale due to pumping is comparable to the diffusion time scale in the high-diffusivity model, the results on inclusion of pumping are qualitatively similar to the results of the high-diffusivity model, even when the diffusivity is assumed to be low.

In recent years, a major thrust in the research on solar dynamo has been to understand how irregularities arise in solar cycles. Both the high-diffusivity and low-diffusivity models have been used to study the irregularities. The irregularities in the solar cycle are believed to be caused by the following three mechanisms.

(*a*) Nonlinear effects giving rise to possible chaotic behaviour.
(*b*) Fluctuations in the Babcock–Leighton mechanism for poloidal field generation.
(*c*) Fluctuations in the meridional circulation.

It is beyond the scope of this review to discuss all these three mechanisms in detail. We shall concentrate here only on the last mechanism which is being studied systematically within the last couple of years, making just a few brief remarks on the first two mechanisms. An earlier review by Choudhuri (2012) discusses all the three mechanisms. For a more complete discussion on the fluctuations in the Babcock–Leighton mechanism, see a still earlier review (Choudhuri 2011). Presumably the nonlinearities are responsible for such things as the Gnevyshev–Ohl effect (Charbonneau, Beaubien & St-Jean 2007). Since the Babcock–Leighton mechanism depends on the tilt angles of bipolar sunspot pairs and these tilt angles show a scatter around the mean given by Joy's law, presumably due to the effect of turbulence on the rising flux tubes (Longcope & Choudhuri 2002), we expect some inherent randomness in the Babcock–Leighton mechanism. When predicting the strength of cycle 24, Choudhuri, Chatterjee & Jiang (2007) assumed this to be the main cause of irregularities in the solar cycle. At that time, the important role played by fluctuations in the meridional circulation was not yet appreciated. Presumably the prediction method developed by Choudhuri, Chatterjee & Jiang (2007) will be reliable only if there is no sudden variation in the meridional circulation between the time when the prediction was made and the time when the cycle eventually reaches its peak.

4. Fluctuations in meridional circulation

It is well known that the period of the flux transport dynamo varies roughly as the inverse of the meridional circulation speed. A slower meridional circulation would make

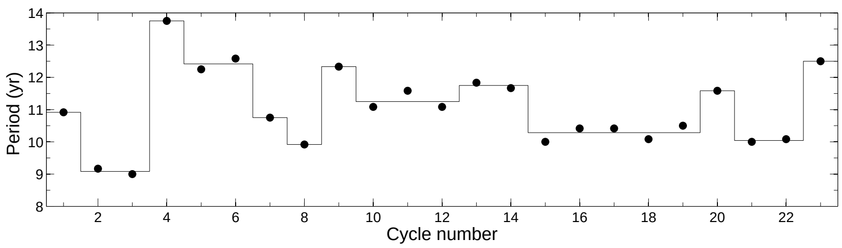

Figure 3. The points show the periods of last 23 solar cycles against cycle number. The solid line is indicative of the trend in variations of period.

the cycle longer. It is, therefore, obvious that fluctuations in meridional circulation would have some effect on the flux transport dynamo. Recently it has been found that the meridional circulation has a periodic variation with the solar cycle, becoming weaker at the time of sunspot maximum (Hathaway & Rightmire 2010; Basu & Antia 2010). Presumably the Lorentz force of the dynamo-generated magnetic field slows down the meridional circulation at the time of the sunspot maximum. Karak & Choudhuri (2012) found that this quenching of meridional circulation by the Lorentz force does not produce irregularities in the cycle, provided the diffusivity is high as we believe. We disagree with the model of Nandy, Muñoz-Jaramillo & Martens (2011) which assumes that the meridional circulation changes randomly at each sunspot maximum. Our point of view is that the periodic variation of meridional circulation due to the Lorentz force cannot be responsible for randomness in solar cycles and we need to consider other kinds of fluctuations in meridional circulation.

We have reliable observational data on the variations of meridional circulation only for a little more than a decade. To draw any conclusion about the variations of meridional circulation at earlier times, we have to rely on indirect arguments. If we assume the cycle period to go inversely as meridional circulation, then we can use periods of different past solar cycles to infer how meridional circulation has varied with time in the last few centuries. Fig. 3 plots periods of several past cycles. We note that several successive cycles had short periods, indicating that the meridional circulation was probably fast at that time. On the other hand, some successive cycles had longer periods, implying a slower meridional circulation. On the basis of such considerations, it appears that the meridional circulation had random fluctuations in the last few centuries with correlation time of the order of 30–40 years (Karak & Choudhuri 2011). We now come to question what effect these random fluctuations of meridional circulation may have on the dynamo. Based on the analysis of Yeates, Nandy & Mckay (2008), we can easily see that high-diffusivity and low-diffusivity dynamos will be affected very differently. Suppose the meridional circulation has suddenly fallen to a low value. This will increase the period of the dynamo and lead to two opposing effects. On the one hand, the differential rotation will have more time to generate the toroidal field and will try to make the cycles stronger. On the other hand, diffusion will also have more time to act on the magnetic fields and will try to make the cycles weaker. Which of these two competing effects wins over will depend on the value of diffusivity. If the diffusivity is high, then the action of diffusivity is more important and the cycles become weaker when the meridional circulation is slower. The opposite happens if the diffusivity is low.

We now address the question if we can compare our theoretical conclusions with any observational data. It has been known for a long time that stronger cycles take shorter time to rise (Waldmeier 1935). In other words, there is an anti-correlation between the

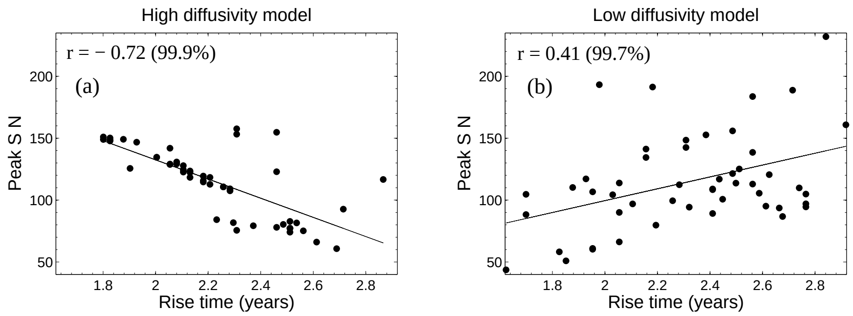

Figure 4. Theoretical plots of correlation between the peak sunspot number and the rise time (in years), obtained by introducing fluctuations in the meridional circulation. The two figures correspond to the high-diffusivity and the low-diffusivity models. Taken from Karak & Choudhuri (2011).

rise times and strengths of cycles. This is known as the Waldmeier effect. The discussion in the last paragraph shows how this effect can be explained with the high-diffusivity dynamo model, which was first done by Karak & Choudhuri (2011). When the meridional circulation slows down, the period becomes longer and simultaneously the cycle becomes weaker in the case of the high-diffusivity dynamo. This means that we would have an anti-correlation between the periods and the cycle strengths. Such an anti-correlation naturally leads also to an anti-correlation between rise times and cycle strengths — the Waldmeier effect. A little reflection will convince you that the opposite — a direct correlation between rise times and cycle strengths — will happen in the low-diffusivity model. Although Charbonneau & Dikpati (2000) considered fluctuations in meridional circulation having much shorter correlation times, still their Fig. 4(c) shows a direct correlation between the cycle duration and cycle amplitude. Charbonneau, Beaubien & St-Jean (2007) made the following comment about the Waldmeier effect: "This has remained notoriously difficult to reproduce within extant solar cycle models." These authors found it so difficult to reproduce the Waldmeier effect because they were using the low-diffusivity model with which it seems impossible to explain the Waldmeier effect. On using the high-diffusivity model, the Waldmeier effect almost falls on your lap, as found by Karak & Choudhuri (2011). Fig. 4 shows correlation plots between rise time and peak sunspot number for both high-diffusivity and low-diffusivity models on introducing fluctuations in meridional circulation. Only the high-diffusivity model produces the Waldmeier effect.

Some of the variations in the cycle strength during the last few cycles seem to be due to fluctuations in meridional circulation. Karak (2010) carried out an interesting simulation in which he varied the meridional circulation to match only the periods of past cycles and found that even the strengths of many cycles got approximately modelled in this process. Fig. 5 taken from Karak (2010) shows the actual sunspot number by the solid line, whereas the dashed line is the sunspot number obtained from the theoretical model by varying the meridional circulation to match the periods. On carrying out the same exercise with the low-diffusivity model, Karak (2010) found that the cycle strengths are not matched at all. Even in the case of the high-diffusivity model, we do not expect the cycles to be matched completely on varying the meridional circulation because fluctuations in the Babcock–Leighton mechanism also contribute to the irregularities of the cycles.

One of the most intriguing aspects of the solar cycle is grand minima when several

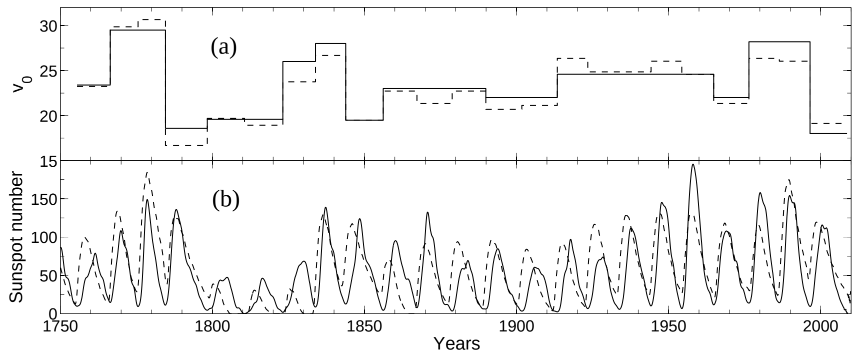

Figure 5. The top panel shows the variation of the meridional circulation amplitude v_0 (in m s^{-1}) obtained from the periods of the past cycles by assuming the period to go as $\propto v_0^{-0.696}$ (dashed line), whereas the solid line indicates v_0 actually used in the theoretical simulations to get the best fit for periods of past cycles. The bottom panel shows the theoretically calculated sunspot number (dashed line) along with the observational sunspot number (solid line). Taken from Karak (2010).

cycles go missing. Although the only well-documented grand minimum to occur after telescopic observations began is the Maunder minimum during 1640-1715, indirect proxies provide evidence for 27 such grand minima in the last 11,000 years (Usoskin, Solanki & Kovaltsov 2007). Since the slowing of the meridional circulation in the high-diffusivity model makes the cycles weaker, one crucial question is whether the Sun can be pushed into a grand minimum if the meridional circulation becomes sufficiently weak due to its intrinsic fluctuations. Karak (2010) showed that this is indeed possible. However, a grand minimum can also occur when the poloidal field falls to very low values due to fluctuations in the Babcock–Leighton mechanism (Choudhuri & Karak 2009). To study the occurrence of grand minima and to calculate its probability, it is necessary to consider fluctuations in the meridional circulation and in the Babcock–Leighton mechanism simultaneously. This has now been done by Choudhuri & Karak (2012). Since the theory of grand minima is discussed in another paper in this Proceedings volume (Karak & Choudhuri 2013), we do not get into the details of this subject here and refer the reader to this other paper.

5. Conclusion

Although several full simulations of the solar dynamo are currently under way, the kinematic mean-field models so far remain the primary theoretical tools for explaining different aspects of the solar cycle in detail. The most promising kinematic mean-field model at the present time is the flux transport dynamo model, in which the toroidal field is generated by the differential rotation at the bottom of the convection zone, the poloidal field is generated near the solar surface by the Babcock–Leighton mechanism and the meridional circulation plays a crucial role by ensuring that magnetic fields are transported in the appropriate directions. In the class of models in which the diffusivity is assumed low, the meridional circulation turns out to be also the primary mechanism for transporting the poloidal field from the surface to the bottom of the convection zone. On the other hand, in the other class of models with high diffusivity, it is the diffusivity which causes the transport of the poloidal field from the top to the bottom. We have

argued that the high-diffusivity flux transport dynamo model is consistent with various aspects of the observational data and is most probably the appropriate model for the solar cycle.

Modelling irregularities of the solar cycle is an active area of research right now. There are three primary mechanisms which may give rise to the irregularities. We have made some comments about two of these mechanisms, nonlinear effects and fluctuations in the Babcock–Leighton mechanism, but have refrained from discussing them in detail due to the limitation of space and also because they have been discussed elsewhere. The third mechanism, fluctuations in the meridional circulation, which is being studied systematically only during the last couple of years, is discussed more fully. Introducing these fluctuations in the high-diffusivity model, we are able to explain such things as the Waldmeier effect. We believe that the fluctuations in the Babcock–Leighton mechanism and in the meridional circulation jointly give rise to the various irregularities of the solar cycle, including the grand minima.

Acknowledgments

My participation in IAU Symposium 294 was made possible by a JC Bose Fellowship awarded by Department of Science and Technology, Government of India.

References

Babcock, H. W. 1961, *ApJ*, 133, 572
Basu, S., & Antia, H. M. 2010, *ApJ*, 717, 488
Brown, B. P., Browning, M. K., Brun, A. S., Miesch, M. S., & Toomre, J. 2010, *ApJ*, 711, 424
Chakraborty, S., Choudhuri, A. R., & Chatterjee, P. 2009, *Phys. Rev. Lett.*, 102, 041102
Charbonneau, P., Beaubien, G., & St-Jean, C. 2007, *ApJ*, 658, 657
Charbonneau, P., & Dikpati, M. 2000, *ApJ*, 543, 1027
Chatterjee, P., & Choudhuri, A. R. 2006, *Solar Phys.*, 239, 29
Chatterjee, P., Nandy, D., & Choudhuri, A. R. 2004, *A&A*, 427, 1019
Choudhuri, A. R. 1989, *Solar Phys.*, 123, 217
Choudhuri, A. R. 1998, *The Physics of Fluids and Plasmas: An Introduction for Astrophysicists* (Cambridge University Press, Cambridge)
Choudhuri, A. R. 2011, In IAU Symp. 273: Physics of Sun and Star Spots (eds. D. P. Choudhury & K. G. Strassenmeier), p. 28
Choudhuri, A. R. 2012, In IAU Symp. 286: Comparative Magnetic Minima: Characterizing quiet times in the Sun and Stars (eds. C. H. Mandrini & D. F. Webb), p. 350
Choudhuri, A. R., Chatterjee, P., & Jiang, J. 2007, *Phys. Rev. Lett.*, 98, 131103
Choudhuri A. R., & Gilman P.A. 1987, *ApJ*, 316, 788
Choudhuri, A. R., & Karak, B. B. 2009, *RAA*, 9, 953
Choudhuri, A. R., & Karak, B. B. 2012, *Phys. Rev. Lett.*, 109, 171103
Choudhuri, A. R., Schüssler, M., & Dikpati, M. 1995, *A&A*, 303, L29
Dikpati, M., & Gilman, P. A. 2006, *ApJ*, 649, 498
D'Silva, S., & Choudhuri, A. R. 1993, *A&A*, 272, 621
Durney, B. R. 1995, *Solar Phys.*, 160, 213
Fan, Y., Fisher, G. H., & DeLuca, E. E. 1993, *ApJ*, 405, 390
Ghizaru, M., Charbonneau, P., & Smolarkiewicz, P. K. 2010, *ApJ*, 715, L133
Gilman, P. A. 1983, *ApJS*, 53, 243
Glatzmaier, G., & Roberts, P. H. 1995, *Nature*, 377, 203
Goel, A., & Choudhuri, A. R. 2009, *RAA*, 9, 115
Hathaway, D. H. 2012, *ApJ*, in press
Hathaway, D. H., & Rightmire, L. 2010, *Science*, 327, 1350
Hotta, H., & Yokoyama, T. 2010, *ApJ*, 714, L308

Jiang, J. 2013, these proceedings.
Jiang, J., Cameron, R. H., Schmitt, D., & Isik, E. 2012, *ApJ*, submitted.
Jiang, J., Chatterjee, P., & Choudhuri, A. R. 2007, *MNRAS*, 381, 1527
Karak, B. B. 2010, *ApJ*, 724, 1021
Karak, B. B., & Choudhuri, A. R. 2011, *MNRAS*, 410, 1503
Karak, B. B., & Choudhuri, A. R. 2012, *Solar Phys.*, 278, 137
Karak, B. B., & Choudhuri, A. R. 2013, these proceedings (arXiv:1211.0165).
Karak, B. B., & Nandy, D. 2012, *ApJ*, in press (arXiv:1206.2106).
Leighton, R. B. 1969, *ApJ*, 156, 1
Longcope, D. W., & Choudhuri, A. R. 2002, *Solar Phys.*, 205, 63
Miesch, M. S., Featherstone, N. A., Rempel, M., & Trampedach, R. 2012, *ApJ*, 757, 128
Nandy, D., & Choudhuri, A. R. 2002, *Science*, 296, 1671
Nandy, D., Muñoz-Jaramillo, A., & Martens, P. C. H. 2011 *Nature* 471, 80
Parker, E. N. 1955, *ApJ*, 122, 293
Parker, E. N. 1979, *Cosmical Magnetic Fields* (Oxford University Press, Oxford)
Steenbeck, M., Krause, F., & Rädler, K. H. 1966, *Z. Naturforsch.*, 21, 369
Usoskin, I. G., Solanki, S. K., & Kovaltsov, G. A. 2007, *A&A*, 471, 301
Waldmeier, M. 1935, *Mitt. Eidgen. Sternw. Zurich*, 14, 105
Wang, Y.-M., Sheeley, N. R., & Nash, A. G. 1991, *ApJ*, 383, 431
Yeates, A. R., Nandy, D., & Mackay, D. H. 2008, *ApJ*, 673, 544
Yoshimura, H. 1975, *ApJ*, 201, 740